New Trends in Applied Geometry

Workshop in Bad Herrenalb, Germany
February 14-19, 2010

Goal

This meeting is intended to bring together a small number of leading research groups in Applied Geometry (i.e. geometric design, geometry processing and discrete differential geometry) including a senior researcher and up to two advanced students of each group. Each young researcher has a one-hour slot for a presentation and an extended discussion. The program will leave room for a lively exchange of new ideas and excellent networking opportunities for the upcoming generation of researchers in the field. Participation is by invitation only.

Location and directions

The workshop takes place at the

Haus der Kirche - Evangelische Akademie Baden
Dobler Str. 51
D-76332 Bad Herrenalb, Germany
Tel ++49-7083-928-0
http://www.ev-akademie-baden.de/haus/hdk-eng.htm
http://www.ev-akademie-baden.de/fr_haus03.htm (German page with more information)

Bad Herrenalb is located nicely in the Black Forest and can be reached conveniently via Frankfurt or Stuttgart airport.
Trains go from there to Karlsruhe, central station.
In front of the station, the local train S1 takes you to Bad Herrenalb.
Sundays, it leaves hourly till 0:17 and arrives 38 minutes later.
The academy lies 1km away from the station in Bad Herrenalb.
Follow the signs, take a taxi or the bus 116 with destination "Oberes Gaistal".
Sundays, the bus leaves bi-hourly till 17:04.

Accomodation and costs

WLAN is available in the lobby and part of the rooms.
Accomodation and all meals for the complete workshop cost 359 Euros.
It has to be paid at the academy, which accepts credit cards.
There is no registration fee and no funding either.

Participants who arrive after dinner on Sunday or attend only part of the workshop pay less if they inform Hartmut Prautzsch till February 7.
There are few double rooms which cost about 5€ less per night and person.
If interested ask soon, since other groups may take them before.

It is possible to stay till Saturday, but not necessarily in the same room and building.
This costs 55-65€ extra.

Organisation

Ulrich Reif, reif@mathematik.tu-darmstadt.de
Hartmut Prautzsch, prautzsch@kit.edu

Program

The workshop starts Sunday, Feb. 14, at 18:30 with the dinner
and ends after lunch on Friday Feb. 19.

We plan
 3 talks every morning (Mo-Fr) and
 2 talks plus discussion sessions for Monday, Tuesday and Thursday afternoon.

The schedule and title of the talks will be announced later.

Nothing is planned for Wednesday afternoon.
Depending on the (possibly snowy) weather, we could

- hike in the mountains (from 400 up to 900m),
- relax in the thermal springs (15 minutes walking distance),
- visit the tiny museum of roof tiles in Bad Herrenalb,
- go to Karlsruhe (50 minutes away by train),
- …

Participants

Nira Dyn
 Kels Shay
 Nir Sharon

Mike Floater
 Christian Schulz

Ron Goldman
 Xiaohong Jia

Jens Gravesen
 Dang Manh Nguyen,
 Peter Nørtoft Nielsen

Kai Hormann,
 Jens Drieseberg
 Tim Winkler

Leif Kobbelt
 David Bommes
 Marcel Campen

Jörg Peters

Konrad Polthier
 n.n.
 n.n.

Hartmut Prautzsch
 Qi Chen

Ulrich Reif
 Nicole Lehmann
 Jennifer Prasiswa

Malcolm Sabin
 Ursula Augsdörfer
 Tom Cashman

Johannes Wallner
 Christian Müller

Ursula Augsdörfer	Group Sabin	Monday, 9:00 – 11:00

Artifacts in Box-spline and Subdivision Curves and Surfaces

Artifacts are present in subdivision and box-spline surfaces. An artifact is a feature of the limit surface which cannot be avoided by movement of control points by the designer. Ideally, the difference between designer intent and what emerges as a limit surface should be eliminated. The first step to achieving this is by understanding and quantifying the artifact observed in the limit surface. Utilizing the subdivision process as a tool for analysis we develop a generic expression to determine the magnitude of artifacts in the limit surface. Our results provide a measure of artifacts with respect to initial control point sampling for curves and surfaces.

David Bommes	Group Kobbelt	Thursday, 9:00 – 11:00

Mixed-Integer Quadrangulation

We present our Mixed-Integer Quadrangulation approach which is very general and enables the complete range from a fully automatic up to a predominantly user guided quadrangulation method. After constructing an as smooth as possible symmetric cross field satisfying a sparse set of directional constraints (to capture the geometric structure of the surface), the mesh is cut open in order to enable a low distortion unfolding. Then a seamless globally smooth parametrization is computed whose iso-parameter lines follow the cross field directions. In contrast to previous methods, sparsely distributed directional constraints are sufficient to automatically determine the appropriate number, type and position of singularities in the quadrangulation. Both steps of the algorithm (cross field and parametrization) can be formulated as a mixed-integer problem which we solve very efficiently by an adaptive greedy solver. Additional user guidance like e.g. prescribed singularity positions can be easily incorporated through linear constraints which are handled by the mixed-integer solver. We show several complex examples where high quality quad meshes are generated.

Marcel Campen	Group Kobbelt	Thursday, 9:00 – 11:00

Hybrid Concepts for Efficient, Accurate, and Robust Operations on Polygonal Meshes

Three-dimensional objects are often represented by their boundary for efficiency. The most common of these boundary representations is the polygon mesh, that specifies a surface in piecewise linear form. The general disadvantage of such, especially piecewise, boundary representations: operations that require changes in outer topology (e.g. Booleans, Repair, Offsetting) are hard to perform robustly and efficiently. This is due to the fact that these changes have to be handled explicitly, e.g. by intersecting and clipping of polygons, appropriately changing mesh connectivity, etc. Due to a high number of special cases that have to be considered and due to the challenges that arise from inconsistencies caused by rounding errors, robustness is hard to achieve without significantly suffering efficiency. One common way to circumvent these problems is to rasterize the objects into discrete volumetric models and perform the desired operations on them. While this usually removes the need to specifically attend topological changes and thus allows for full robustness quite efficiently, it naturally limits the accuracy of the result considerably. We developed two novel approaches to solve these problems concerning robustness, accuracy, and efficiency. The first approach builds upon the paradigm of hybrid geometry representation and processing. By combining an explicit boundary representation with a discrete volumetric representation, one is able to exploit the geometrical accuracy of the former one and combine it with the topological simplicity of the latter one. In this way efficient robust algorithms can be obtained, that increase the quality of the result over traditional purely volumetric approaches. The second approach carries this concept to the extreme: boundary and volumetric representations are not just combined but really fused. Basically, voxels are no longer considered rectilinear, but are shaped arbitrarily. By this means they can match up to the geometry of the polygonal boundary representation perfectly and in the end allow us to obtain fully accurate results. By employing a special polygon representation and hierarchical decomposition techniques in this context, operations can still be performed robustly without the overhead of slow arbitrary precision arithmetics that is usually to be accepted for this goal.

Tom Cashman	Group Sabin	Monday, 9:00 – 11:00

Subdivision Surfaces and the Dolphin Problem

There is an open problem in Computer Vision which this talk names the ‘dolphin problem’. The challenge is to start from multiple photographs of objects from the same class (such as dolphins), where each image sees a different individual in a different pose. This type of collection is typically the result of searching for images that match a certain keyword, for example. The dolphin problem asks whether we can use these images to learn a deformable surface model for objects of the photographed class, with the pose and view parameters for each individual. This involves an extension of existing surface reconstruction techniques to both multiple poses and multiple views, but the problem is essentially geometric. For example, a formulation of the problem without pose variation is: can we create a fair surface with specified silhouette shapes, where the silhouettes are not necessarily planar cross-sections from the surface? I will describe an attempt to tackle the dolphin problem using subdivision surfaces as an underlying representation, considering both the challenges and benefits of this strategy. This work was the outcome of an internship at Microsoft Research, Cambridge, in collaboration with Andrew Fitzgibbon.

Qi Chen	Group Prautzsch	Wednesday, 11:20 – 12:20

A Geometric Framework for Smoothness Analysis of General Classes of Subdivision Algorithms

To analyze the smoothness of a subdivision algorithm at extraordinary points, we often use Reif's C^1-criterion by checking the regularity and injectivity of the characteristic map of this subdivision algorithm for every valence of the extraordinary points. To analyze the smoothness of a whole class of subdivision algorithms, it is usually complicated or even unrealistic if we use traditional numerical methods to study the characteristic map for every subdivision algorithm variant. In this talk, I develop a geometric framework for analyzing general classes of (symmetric) subdivision algorithms that can be factorized into simple convex combination operators. In particular, I first discuss how to analyze the characteristic maps of all midpoint subdivision schemes as well as the spectral properties of their subdivision matrices. Second, I study a wider class of subdivision schemes which includes simplest and midpoint subdivision schemes and all their possible combinations. I show that these schemes except for a few are C^1-schemes.

Jens Drieseberg	Group Hormann	Thursday, 15:00 – 17:00

Thinning Mesh Animations

In the first part of the talk we present a method for thinning mesh animations. The main idea is to pre-process 3D animation sequences and to reduce the number of frames before compressing them with an arbitrary state-of-the-art compression scheme. Our method is based on Batch Neural Gas, a clustering and classification approach that can be used to automatically find the most relevant frames from the sequence. The meshes from the original sequence can then be expressed as linear combinations of these few key-frames with small approximation error. Once the key-frames are found, they cannot only be used to reconstruct the whole sequence, they are also suitable for mesh interpolation and extrapolation to generate new poses, thus allowing for easy exploration of the shape space spanned by the key-frames. The second part of the talk is about interpolation problems that arise when the input meshes differ by large rotations. We present an approach to solve the problems by using a traverse order to correct the rotation angles and axes and the difficulties and limits of this method.

Klaus Hildebrandt	Group Polthier	Friday, 11:20 – 12:20

Fairing and Spectral Mesh Processing

Fairing and Spectral Mesh Processing is discussed.

Xiaohong Jia	Group Goldman	Thursday, 11:20 – 12:20

Set-Theoretic Generators of Rational Space Curves

We show how to calculate three low degree set-theoretic generators for all rational space curves of low degree (degree ≤ 6) as well as for all higher degree rational space curves where at least one element of their μ-basis has degree 1 from a μ-basis of the parametrization. In addition to having low degree, at least two of these surface generators are always ruled surfaces. Whenever possible we also show how to compute two set-theoretic complete intersection generators for these rational space curves from a μ-basis of their parametrization.

Nicole Lehmann	Group Reif	Wednesday, 9:00 – 11:00

Surface Smoothing via the Embedded Weingarten Map

The embedded Weingarten map is an extension of the standard shape operator from the tangent space to the embedding space of a surface. It is a geometric invariant of second order containing complete curvature information. The defining equation can be easily discretized so that the embedded Weingarten map can be used to estimate curvature of a facetted surface in an efficient and robust way. As an application, we show results of isotropic surface smoothing based on the new approach.

Christian Müller	Group Wallner	Monday, 11:20 – 12:20

Discrete Conformality and Discrete Minimal Surfaces

We explore notions of discrete and semi-discrete conformality. In the discrete setting we consider hexagonal meshes with planar faces. In particular we introduce the notion of conformal hexagon and study discrete surfaces consisting of such hexagons. This analogue of a conformal parametrization leads to a Christoffel-type transformation and a construction of discrete minimal surfaces. In the semi-discrete setting we study surfaces which consist of ruled surface strips. For semi-discrete surfaces we consider conformal equivalence, dualizability, and incidence-geometric characterizations of dualizability. We also construct semi-discrete minimal surfaces

Dang Manh Nguyen	Group Gravesen	Friday, 9:00 – 11:00

Isogeometric Design of Vibrating Membranes

The problem of designing vibrating membranes was first considered by J.W. Hutchinson and F.I. Niordson (Continuum mechanics and related problems of analysis, 1972). In particular, they considered the problem of the design of a harmonic drum, i.e., a membrane where the eigenvalues of the Laplace operator are in harmonic proportion. This is related to M. Kac's famous question "Can one hear the shape of a drum?" (AMM, 1966). Later, C. Kane and M. Schoenauer (CI, 1995) have attacked the problem by genetic algorithms. We propose to use isogeometric analysis and shape optimization to solve the problem. More precisely, we want to find the shape of a membrane whose first few eigenvalues are prescribed. As the eigenvalues do not determine the shape of the membrane (see G. Gordon, D. L. Webb, and S. W. Wolpert, BAMS, 1992) we minimize the length of the perimeter under constraints on the first few eigenvalues. The main result is a successful application of isogeometric analysis in the area of shape optimization. An interesting and challenging example is the case of the harmonic drum. Here we want the first four eigenvalues to satisfy the equations $\lambda_1 = \frac{4}{9}\lambda_2 = \frac{4}{9}\lambda_3 = \frac{1}{4}\lambda_4$. Eigenvalues are not differentiable functions when their multiplicities change so the double eigenvalue $\lambda_2 = \lambda_3$ poses a problem for the optimization. We have two strategies for overcoming this problem. The first is to change the problem slightly so the the two eigenvalues are not exactly equal but separated by a small amount. Another way is to look at the functions $\lambda_1 + \lambda_2$ and $\lambda_1\lambda_2$ which are differentiable. B-splines of different orders and geometrically modeling methods of both a single patch and three identical patches ensuring exact 120 degree symmetry have been used. Furthermore, we have tried different methods to obtain the inner parametrization. We obtained consistent and satisfactory results of the optimization. The results from this work suggest that isogeometric analysis provides an effective design tool which is capable of designing models with high degree of boundary continuity and high accuracy.

Peter Nortoft Nielsen	Group Gravesen	Friday, 9:00 – 11:00

Isogeometric Analysis and Shape Optimization in Stokes Flow Problems

Isogeometric analysis unites the power to analyse complex engineering problems from finite element analysis (FEA) with the ability to represent complicated shapes from computer aided design (CAD). Being a mixture of CAD and FEA, isogeometric analysis serves as an ideal basis for shape optimization. The aim of this work is to apply this to 2-dimensional, steady-state Stokes flow problems with Dirichlet boundary conditions. In the first part of the talk we give an introduction the isogeometric analysis of this type of problems. This is done by firstly stating the governing equations in the strong form of the problem. Then the geometry parametrization based on B-splines is introduced, after which the isogeometric approach of Galerkin's numerical method is described. Different isogeometric implementations are then tested numerically for stability, and finally a simple error convergence study is presented. The second and final part of the talk deals with isogeometric shape optimization of the considered type of problems. We present implementation details and results for a simple optimization problem in which the objective is to find an optimal shape a 2-dimensional pipe bend that minimizes the pressure drop from the inlet to the outlet, subjected to a constraint on the total area of the pipe.

Jörg Peters	Group Peters	Tuesday, 11:20 – 12:20

Reparameterizations and Boundary Data in Surface Construction

Some new insights into some old challenges:
1. minimal number and multiplicity of knots of any C^1 and C^2 free-form spline constructions
2. lowest-degree C^1 spline constructions
3. a new geometric characterization of the vertex enclosure constraint
4. an open problem in this regard.

Jennifer Prasiswa	Group Reif	Wednesday, 9:00 – 11:00

Scattered Data Fitting and Hole-Filling Using Extended B-Splines

Given scattered data (ξ_i, f_i) on a domain $\Omega \subset \mathbb{R}^d$, standard splines fitting techniques either yield shape artifacts near the boundary or require a delicate choice of weights for fairness functionals. In this talk, we present new methods based on extended B-splines which combine good shape properties with optimal error estimates, and avoid the use of artificial smoothing terms. For problems of modest size a global weighted least squares fit is suitable, while for large problems a two-stage method combining local approximations is favorable in order to reduce the computational expense. If the data $f_i = f(\xi_i)$ are sampled from a smooth function and are sufficiently dense, then the use of extended B-splines of order n yields the bound $|\Delta|_{W_p^m(\Omega)} \leq c\ h^{n-m}|f|_{W_p^n(\Omega)}$ on the approximation error Δ. Numerical experiments illustrate the potential of these methods and validate the error estimate. As a possible application for these algorithms we present an iterative algorithm for Hole-Filling.

Christian Schulz	Group Floater	Monday, 15:00 – 17:00

From Barycentric Coordinates to Transfinite Hermite Interpolation

In the last 10 to 15 years there has been an increased research interest in barycentric coordinates. Starting with the mean value coordinates for star shaped polygons, new coordinates and extensions of them for general polygons are now being used in applications like, amongst others, mesh and/or image deformation and image cloning. Moreover, barycentric coordinates have been extended to generate smooth transfinite interpolants for the Lagrange and recently also for the Hermite case. The latter was achieved by a method called pointwise radial minimization (PRM). In this presentation we will give a rough overview over parts of this development, present pointwise radial minimization in more detail and discuss related open questions and possible future research ares.

Nir Sharon	Group Dyn	Tuesday, 9:00 – 11:00

Interpolation of Parametric Curves by Surfaces Based on Univariate Subdivision

In this talk we present a method for the interpolation of a bivariate function from its values along parallel lines. This method can be applied to the construction of a surface passing through a given set of parametric curves. Following the construction of polyspline and tension surfaces, we reduce the bivariate interpolation problem to a countable number of univariate interpolation problems. The univariate interpolation problems are solved by univariate, interpolatory, non-stationary subdivision schemes. We present properties of the generated interpolant such as continuity, smoothness and approximation order. We conclude the talk with numerical simulations of the method.

Kels Shay	Group Dyn	Tuesday, 9:00 – 11:00

Interpolatory Subdivision of Sets with Applications to Reconstruction of objects from cross-sections

We introduce a new geometric definition of affine combinations of two compact subsets of $\mathbb{R}^n$. The 4-point interpolatory subdivision schemes for points is adapted to sets, by first expressing the insertion rule in terms of repeated binary averages, and then replacing these affine combinations of pairs of points by the new affine combinations of sets. This subdivision scheme is applied to the reconstruction of multidimensional objects from cross-sections. We address the questions of convergence and approximation order of this new set-valued subdivision scheme and present several computational examples.

Tim Winkler	Group Hormann	Thursday, 15:00 – 17:00

Mesh Interpolation and Optimization

Interpolating vertex positions among triangle meshes with identical vertex-edge graphs is a fundamental part of many geometric modelling systems. Linear vertex interpolation is robust but fails to preserve local shape. Most recent approaches identify local affine transformations for parts of the mesh, model desired interpolations of the affine transformations, and then optimize vertex positions to conform with the desired transformations. However, the local interpolation of the rotational part is non-trivial for more than two input configurations and ambiguous if the meshes are deformed significantly. We propose a solution to the vertex interpolation problem that starts from interpolating the local metric (edge lengths) and mean curvature (dihedral angles) and makes consistent choices of local affine transformations using shape matching applied to successively larger parts of the mesh. The local interpolation can be applied to any number of input vertex configurations and due to the hierarchical scheme for generating consolidated vertex positions, the approach is fast and can be applied to very large meshes. In the second part of the talk we show an approach that allows us to create compatible input meshes for the aforementioned interpolation method and to smooth the vertex-paths during interpolation. On the one hand, the framework allows us to control the shapes of the mesh triangles by applying simple averaging operations, on the other we can control the Hausdorff distance to some reference geometry by minimizing a quadratic energy. Due to the simplicity of this setup, the framework is efficient and easy to implement. Yet it constitutes an effective and versatile tool with a variety of possible applications. Due to the flexibility of the framework, it can be applied in a lot more scenarios. We show how to use it to reduce the texture distortion in animated mesh sequences, to improve the results of cross-parameterizations, and to minimize the distance between meshes and their remeshes.